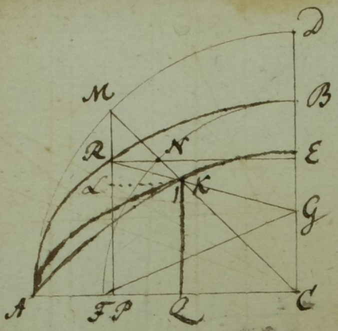


Sit $AC = CD = 1$.

CB = sin. elevat. poli = cos e

CF = cos. elev. poli = sin e.

MCD = ang. horario = ω

GFC = ang. declin = d.

ductis CM, MP, NF erit

RGE azimuthum

et ARB ellipsis, cuius focus F.

Porro ducta RG, erit K in sectione conica, KCG ang. horarius

KGB azimuthum, C focus huius curvae AEK, EC axis maior.

Demonstratio

Sit $MC = 1$, $MN = a$, $CG = c$, $MCA = \omega$,

$CQ = x$, $QK = y$

erit

$MR = a \cos \omega$.

$CP = \sin \omega$.

$MP = \cos \omega$. $\quad x : y = \sin \omega : \cos \omega$.

$MR : CG = PQ : QC = ML : KQ$

$a . \cos \omega : c = (\cos \omega - y) : y = (\sin \omega - x) : x$

$ay \cos \omega = c \cos \omega - cy$

$\cos \omega = cy : (c - ay)$

$\cos \omega^2 = c^2 y^2 : (c - ay)^2$

$\sin \omega^2 = 1 - c^2 y^2 : (c - ay)^2$

$\frac{\cos \omega^2}{\sin \omega^2} = c^2 y^2 : (c^2 - 2cay + a^2 y^2 - c^2 y^2) = y^2 : x^2$

adeoque

$c^2 x^2 = c^2 - 2cay + a^2 y^2 - c^2 y^2$.

Quae aequatio est ad Sectionem conicam.

Quodsi fuerit $a = c$, erit $cx^2 = c - 2ay$, quare hoc casu curva AKE erit parabola.

Si ponatur $c > a$, erit

$$y^2 + \frac{2ac}{c^2-a^2}\cdot y + \frac{cc}{c^2-a^2} = \div\frac{c^2x^2}{c^2-a^2}$$

Ponatur $x = 0$, erit

$$y^2 + \frac{2ac}{c^2-a^2}\cdot y = \frac{cc}{c^2-a^2}$$

unde

$$y = -\frac{ac}{cc-aa} \pm \sqrt{\frac{cc}{cc-aa}}$$

sive

$$y = \frac{(-a \pm c)c}{cc-aa} = \frac{c}{c+a} \text{ vel } -\frac{c}{c-a}$$

Quae ergo aequatio est ad ellipsin, cuius

semiaxis maior $= \frac{cc}{cc-aa}$

semiaxis minor $= \sqrt{\frac{cc}{cc-aa}}$

distantia foci a centro $= \frac{ac}{cc-aa}$

dist. foci a vertice $= \frac{cc \pm ac}{cc-aa} = \frac{c}{c \mp a}$

Quare centrum ellipseos ARB est focus ellipseos AKE. et ordinata AC foco insistit.

Est vero $c = \sin e . \tan d.$

$a = 1 - \cos e.$

unde

$$c + a = 1 - \cos e + \sin e \tan d = \frac{\cos d - (\cos e \cos d - \sin e \sin d)}{\cos d} = \frac{\cos d - \cos(e+d)}{\cos d}$$

$$= \frac{2\sin(\frac{1}{2}e + d) . \sin\frac{1}{2}e}{\cos d}$$

$$c - a = \sin e \tan d - 1 + \cos e = \frac{\sin e \sin d + \cos e \cos d - \cos d}{\cos d} = \frac{\cos(e-d) - \cos d}{\cos d}$$

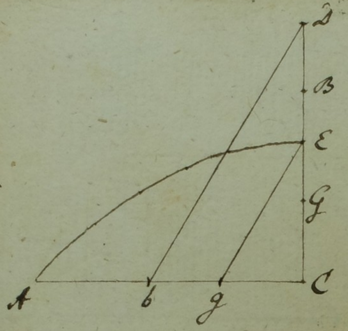
$$= \frac{2\sin\frac{1}{2}e . \sin(d - \frac{1}{2}e)}{\cos d}$$

Quare ubi $x = 0$, erit

$$y = \frac{c}{c+a} = \frac{\sin e . \tan d . \cos d}{2\sin(\frac{1}{2}e + d) . \sin\frac{1}{2}e} = \frac{\sin d . \cos\frac{1}{2}e}{\sin(\frac{1}{2}e + d)}$$

$$\dot{y} = \frac{-c}{c-a} = \frac{-\sin e . \tan d . \cos d}{2\sin\frac{1}{2}e . \sin(d - \frac{1}{2}e)} = \frac{\div \sin d . \cos\frac{1}{2}e}{\sin(d - \frac{1}{2}e)}$$

$$ca = \sin e \tan d - \cos e \sin e \tan d . = 2\sin\tfrac{1}{2}\tilde{e} . \tan d.$$



Cum sit $y = \frac{c}{c+a} = CE$

fiat $AC = 1. = CD$

$CB = \cos e.$

$CG = Cg = \text{se}.\text{td}.$

$gb = DB = 1 - \cos e$

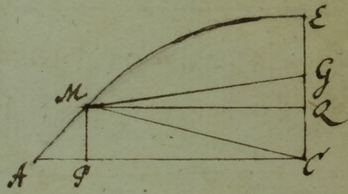
agatur bD ipsi gE parallela

erit E vertex, C focus, et AC ordinata focalis.

Quodsi fuerit $c = a$, ~~[illegible]~~ curva erit parabola, quod fit ubi $d = \frac{1}{2}e$. Hocque casu est $CE = \frac{1}{2} = \frac{1}{2} AC$.

Quodsi fuerit $c = CB = 1 - a$, erit $CE = c = CB$. Quare g et E in B coincidunt, quod vel per se clarum, quia sidus per verticem transit.

Si fuerit $d = e$, sidus inoccid[u]uum est, et horizontem attingit.



Solutio Problematis analytica.

Detur $GC = g$

azimuthum $MGC = a$.

ang. horarius $MCG = \omega$

compl. declin. $= c$

elev. aequatoris $= e$.

Quaerit[ur] curva AME. Est vero

$$\sin e . \cot c = \cos e . \cos \omega + \sin \omega . \cot a.$$

Fiat $PC = x$, $PM = y$ erit

$$\cot a = \frac{GQ}{MQ} = \frac{g-y}{x}.$$

$$\sin \omega = x : \sqrt{x^2+y^2} \quad \cos \omega = y : \sqrt{x^2+y^2}$$

Quare

$$\text{se}.\cot c = \frac{x \cos e . y + x.(g-y)}{x . \sqrt{(x^2+y^2)}} = \frac{\cos e . y + g - y}{\sqrt{x^2+y^2}}$$

unde

$$(x^2+y^2)\, \text{se}^2 \cot c^2 = (\cos e . y + g - y)^2 = [g - (1-\cos e) y]^2$$

~~[illegible]~~

Unde

$$x^2 \sec^2 \cot c^2 = g^2 - 2g(1-\cos e)y + (1-\cos e)^2 y^2 - y^2 \sec^2 \cot c^2.$$

Cum $y = 0$, erit

$$x = \frac{g}{\sec \cot c} = AC.$$

Ponatur $AC = 1$. erit $g = \sec \cot c$.

fiatque $1 - \cos e = a$.

erit

$$x^2 g^2 = g^2 - 2gay + (a^2 - g^2) y^2.$$

Quae eadem est aequatio quam supra habuimus.

Cum sit $g\sqrt{(x^2+y^2)} = g - ay$

erit

$$\sqrt{x^2+y^2} = 1 - \frac{a}{g} y = 1 - \frac{1-\cos e}{\sec \cot c} y = 1 - \frac{tc}{\cos \frac{1}{2} e} \cdot y$$

Est vero

$$\frac{y}{\sqrt{x^2+y^2}} = \frac{y}{1 - \frac{tc}{\cos 2e} y} = \cos \omega.$$

j-m.moebs@melpost.fr.